1. Obtain the compatibility equation in terms of stress components for a 2-D problem of elasticity when there are no body forces. Hence obtain the general 3rd order polynomial solution for this differential equation and describe the physical stress state it depicts.

2. Evaluate the stresses and displacements for a cantilever loaded at the free end.

3. Explain stress ellipsoid and stress invariants. Evaluate the principal stress, both direct and shear, and the principal planes if the stress at a point is given as follows.

\[
\begin{pmatrix}
10 & 2 & 6 \\
2 & 8 & 4 \\
6 & 4 & -6
\end{pmatrix}
\]

4. Using general solution for an axisymmetrical problem in polar coordinates obtain the stresses and displacements in a curved beam subjected to pure bending.

5. Explain the stress concentration that occurs around a hole made in an infinitely large plate. Under a uniform direct stress.

6. Explain the following
   i) Strain components in polar coordinates.
   ii) Homogeneous deformation
   iii) Rotation.

1. Explain plane stress and plane strain problems.
2. What is a strain rosette? And how is it constructed?
4. Give the basic equations of equilibrium and stress-strain for axisymmetric problem neglecting body forces.
5. Explain the phenomenon “Strain Hardening”.
7. (a) Explain the equations of compatibility.
   (b) State the stress and strain transformation laws.

8. Establish the relationship between various constants of elasticity.
9. The state of strain at a point is given by
   \[
   \begin{align*}
   \varepsilon_X &= 0.001 & \varepsilon_Y &= -0.003 & \varepsilon_Z &= 0.002 \\
   \gamma_{XY} &= 0.001 & \gamma_{YZ} &= 0.005 & \gamma_{XZ} &= -0.002
   \end{align*}
   \]

10. Determine the bending stress and shear stress at a section in a cantilever beam with a point loaded at the free end using two dimensional rectangular coordinates.
11. Using Fourier integral method, determine the solution of biharmonic equation in Cartesian coordinates.
12. A semi-infinite elastic medium is subjected to a normal pressure of intensity “p” distributed over a circular area of radius “a” at x = 0. Determine the stress distribution by using Fourier integral.
13. Explain St.Venant’s Theory using a suitable example of torsional problem.
14. Establish the torsional moment carrying capacity of an equilateral triangle cross sectional bar.
15. Explain any three Theories of Failure and give the governing equations. Also explain the limitations of those theories.
16. Explain: (a) Plastic flow (b) Yield surface, and (c) Plastic potential

1.a) Explain Hookes law and then derive stress strain relations.
b) Define a state of (i) plane stress (ii) plane strain and explain stress & strain components. Give examples for each.
c) Write equations of equilibrium, boundary conditions & compatibility equation for 2-D problem of elasticity.
2. Obtain a 4th order polynomial solution for the differential equation in terms of stress function. Hence evaluate stresses and displacements for a cantilever beam loaded at the free end.
3. Derive the differential equation in terms of polar coordinates and obtain a solution for an axisymmetric problem. Obtain stress components in a circular disc with a central hole.
4. Evaluate the effect of a circular hole on stress distribution in plates subjected to uniform normal stress.
5. For a problem of bending of a curved bar by a force at the free end calculate stresses and displacements.

6. Write short notes on
   a) Stress Ellipsoid
   b) Stress invariants
   c) Principal stresses & planes for normal and shear stresses.

   1. Considering as three dimensional problem of elasticity evaluate displacements in a prismatical bar under its own weight.
   2. Explain the difference in behavior of a circular shaft and straight bars under torsion. Hence explain saint venants Semi inverse method. Apply the same to an elliptical cross section and obtain shear stress and displacements in the cross section.
   3. How is membrane analogy applied to a problem of torsion in non-circular shafts, evaluate shear stress in a narrow rectangular section and apply the same to twist in rolled profiled steel sections.
   4. If a cantilever beam is subjected to point load at the free end calculate shear stresses if the cross section is circular.
   5. Explain soap film method
   6. Explain briefly
      i) Torsion of hollow shaft
      ii) Strain energy of bodies
      iii) The principle of superposition
      iv) Failure theories or yield criterion in plastic behavior

   1. Derive the equations of equilibrium in terms of displacements for a 3-D problem of elasticity.
   2. Solve a problem of pure bending of prismatic bar as a 3-D problem of elasticity and obtain the displacements.
   3. Explain membrane analogy .Apply this analogy to the problem of bar of narrow rectangular section subjected to torsion and calculate the stresses.
   4. Explain the difference in behavior of a circular shaft and straight bars under torsion. Hence explain saint venants Semi inverse method. Apply the same to an elliptical cross section and obtain shear stress and displacements in the cross section.
   5. How is membrane analogy applied to a problem of torsion in non-circular shafts, evaluate shear stress in a narrow rectangular section and apply the same to twist in rolled profiled steel sections.

   6. a) Plastic deformation and molecular behaviour of material causing yielding.
      b) write the assumptions and different yield criteria and explain failure theories for elastic material.
   8. Explain membrane analogy for a obtaining behaviour of non circular shafts under torsion.
   9. Calculate shear stresses and twisting moment in a narrow rectangular section. Obtain the same for a rolled profile section.

10. Write short notes on
   a. Soap Film Method
   b. Torsion of thin tubes & Hollow sections

11. Evaluate shear stresses in a rectangular section of a cantilever beam loaded at the free end.

12. Explain the different theories failure and write yield criterion for each.

   1. Explain Saint Venants semi inverse method for evaluation of torsion in prismatic shafts. Hence calculate torsional moment and shear stresses in terms of stress function.
   2. Explain membrane analogy for a obtaining behaviour of non circular shafts under torsion.
   3. Calculate shear stresses and twisting moment in a narrow rectangular section. Obtain the same for a rolled profile section.
4. Write short notes on
   a. Soap Film Method
   b. Torsion of thin tubes & Hollow sections

5. Evaluate shear stresses in a rectangular section of a cantilever beam loaded at the free end.

6. Explain the different theories failure and write yield criterion for each.

1. Explain.
   a) i) Hooke’s law   ii) Compatibility Condition
      iii) Plane stress   iv) Plane strain
   b) Derive the Differential equation of equilibrium based on equilibrium equations. Boundary conditions, compatibility conditions for a 2-D plane stress problem.

2. Obtain a solution for stresses in a cantilever beam with a load at the end using polynomial solution of differential equilibrium equation. Hence also obtain displacements of the beam.

3. Evaluate the stress distribution in a plate subjected to uniform tension in both directions when a small circular hole is made in the middle of the plate.

4. Derive the equations of equilibrium for a 3-D problem of elasticity.

5. Solve a problem of pure bending of prismatic bar as a 3-D problem of elasticity and obtain the displacements.

6. Using Saint Venant semi-inverse method for the problem of Torsion of straight bars derive the solution.

7. Explain membrane analogy. Apply this analogy to the problem of bar of narrow rectangular section subjected to torsion and calculate the stresses.

8. Explain briefly
   (i) Stress invariance
   (ii) Stress ellipsoid
   (iii) Principal stress & principal planes
   (iv) Homogeneous deformation

1.a) Explain Hooke’s law and then derive stress strain relations.
   b) Define a state of (i) plane stress (ii) plane strain and explain stress & strain components. Give examples for each.
   c) Derive the differential equation for a 2-D problem of elasticity in static equilibrium.

2. Evaluate stresses and displacements for a cantilever beam loaded at the free end.

3. Write the differential equation in terms of polar coordinates for an axisymmetric problem. Obtain stress components in a circular disc with a central hole and hence evaluate.

4. Evaluate the effect of a circular hole on stress distribution in plates subjected to uniform normal stress. Hence calculate the stress concentrations in such plate.

5. For a problem of bending of a curved bar by a force at the free end calculate stresses and displacements.

6. Explain from basics
   a) Stress Ellipsoid
   b) Stress invariants and their significance
   c) Principal stresses & planes for normal and shear stresses in 3-D problem.

1.a) Define warping.
   b) Derive the equations for twisting moment and shear stresses in straight bars of non-circular cross sections. Hence evaluate the same for an elliptical cross section.

2. Explain membrane analogy for torsion of prismatic shafts. Hence obtain solution to the problem of torsion. Hence obtain solution to the problem of a bar with narrow rectangular cross section.

3. Explain briefly with relevant equations
   i) Torsion of rolled profile sections
   ii) Torsion of thin tubes
   iii) Torsion of hollow sections

4. Evaluate shear stresses in a cantilever bar with a point load at the force end. Obtain stresses variation in the cross section if the bar is circular in section.

5. a) What is soap film method.
   b) Write the equation of equilibrium for a 3-D problem in elasticity in terms of displacements.
6. a) Derive expression for strain energy and distraction energy.
   b) Define state of plasticity
   c) Explain different theories of failure.

1. a) Obtain the strain displacement relations.
   b) Derive the D.E of equilibrium in plane stress considering body forces.
2. Explain airy’s stress function, investigate the given function is stress function is not.
   \( \Phi = (a\theta + b\theta + c\theta + d\theta) x \) find x.
3. Investigate what problem of plane stress is satisfies by the stress function.
   \( \Phi = 3f/4d \ ( xy - xy^2x^2 ) + p/2 \ y^2 \)
   Applied in the region y = o; y = d; x = o on the ride x positive.
4. Obtain the compatibility equitation is plans strain considering the body forces.
5. a) Explain stress tensor and strain tensor.
   b) The rate of stress at a point with respect xyz plane is
   \[
   \begin{pmatrix}
   10 & 4 & -6 \\
   4 & 5 & -5 \\
   -6 & -5 & 2
   \end{pmatrix}
   \text{kN/mm}^2
   \]
   Determine the stress tensor relation to x'y'z' plane by a rotation through 600 about z – axis.
6. Obtain the stress for a simply supported beam subjected to sinusoidal loading on the upper and lower edges.

1. Obtain the compatibility equation in terms of stress components for a 2-D problem of elasticity when there are no body forces. Hence obtain the general 3rd order polynomial solution for this differential equation and describe the physical stress state it depicts.
2. Evaluate the stresses and displacements for a simply supported beam under uniformly distributed load
3. Using general solution for an axisymmetric problem in polar coordinates obtain the stresses and displacements in a circular disk
4. Apply the general polynomial solution to the problem of curved bar fixed at one end and bending due to a load P applied at the other end. Obtain the deflections at loaded end.
5. Evaluate the principal stress, both direct and shear, and the principal planes if the stress at a point is given as follows.
   \[
   \begin{pmatrix}
   12 & 4 & 2 \\
   4 & 6 & 0 \\
   2 & 0 & -10
   \end{pmatrix}
   \]
6. Explain the following
   i) Strain components in polar coordinates.
   ii) Stress ellipsoid and stress invariants

1. Apply a polynomial solution to the differential equation of equilibrium to the problem of straight bar simply supported and with UDL on the entire span. Obtain the deflections at mid span.
2. From the general solution of symmetric stress distribution problem in polar coordinates derive the stresses in the case of a circular plate with a hole at center?
3. When a curved bar is bending due to force applied at one end, find out the stresses in the c/s and deformation of the bar.
4. Explain membrane analogy. Apply this analogy to the problem of bar of narrow rectangular section subjected to torsion and calculate the stress
5. Derive the saint venants solution to the problem of Torsion in straight bars and apply this solution to a bar with elliptical cross section.
6. What is meant by stress tensor. The state of stress at a point with respect to x-y-z system is
Determine the stress tensor relation to other plane by a rotation through $30^0$.

7. Evaluate the stress distribution and displacements in prismatic bar under its own weight treating it as a 3-D problem.

8. Explain briefly
   a) Torsion of hollow sections
   b) Soap film method

1. Apply a polynomial solution to the differential equation of equilibrium to the problem of straight bar fixed at one end and bending due to a load $P$ applied at the other end. Obtain the deflections at loaded end.
2. From the general solution of symmetric stress distribution problem in polar coordinates derive the stresses in the case of pure bending of curved bar?
3. Evaluate the effect of a circular hole on stress distribution in infinite plate subjected to uniform tension in one direction.
4. Explain the stress distribution in rotating disk and the effect of a hole at the center of disk?
5. Explain membrane analogy. Apply this analogy to the problem of bar of narrow rectangular section subjected to torsion and calculate the stress.
6. Derive the Saint Venants solution to the problem of Torsion in straight bars and apply this solution to a bar with elliptical cross section.
7. Evaluate the stress distribution and displacements in prismatic bar subjected to pure bending treating it as a 3-D problem.

1. Define Hooke’s law and stress strain relations for a deformable body of elastic material. Obtain equilibrium equation and boundary conditions and hence arrive at compatibility condition in term of stress components for a plane stress condition.
2. Evaluate the stress components in the cross section and deformations of a simply supported beam loaded with UDL.
3. Obtain the effect of a circular hole on stress distribution in plates.
4. When a curved bar is bending due to force applied at one end, find out the stresses in the c/s and deformation of the bar.
5. Explain stress ellipsoid and stress invanants Calculate principal stresses for the following stress tensor at a point in a 3-D body.

\[
\begin{pmatrix}
12 & 0 & 6 \\
0 & 10 & 4 \\
6 & 4 & 14
\end{pmatrix}
\]
   b) When a prismatic bar is stretching by its own weight, obtain displacements of bar at the free end.
7. Explain membrane analogy. Apply the same to a bar of narrow rectangular section and evaluate shear stresses in cross section.
8. Explain briefly
   i) Soap film method
   ii) Torsion of rolled profiled section
9. Define Hooke’s law and stress strain relations for a deformable body of elastic material. Obtain equilibrium equation and boundary conditions and hence arrive at compatibility condition in term of stress components for a plane stress condition.

10. Evaluate the stress components in the cross section deformations in a simply supported beam loaded with UDL.

11. Obtain the effect of a circular hole on stress distribution in plates.

12. When a curved bar is bending due to force applied at one end, find out the stresses in the c/s and deformation of the bar
   a) Write equations of equilibrium in term of displacements for 3-D problem of elasticity.

13. Explain membrane analogy. Apply the same to a bar of narrow rectangular section and evaluate shear stresses in cross section.

14. Explain briefly
   i) Soap film method
   ii) Torsion rolled profiled section
   iii) Evaluate the displacements in pure bending of prismatic bar.

7. State and explain saint venants semi inverse method for prismatic bars under torsion. Hence arrive at shear stress and torque values in terms of stress function Ø. Applying the same to a bar of elliptic c/s obtain distribution of shear stress in the c/s and warping displacement in c/s.

8. Derive membrane analogy. Apply this to the torsion of bar of narrow rectangular cross section.

9. Evaluate the shear stress distribution in a cantilever bar of circular cross section, loaded at the free end.


11. Explain
   iii) Torsion of thin tubes
   iv) Failure theories for Elastic / Plastic behavior of materials.

7. Derive the differential equation of equilibrium for 2 – D problem of elasticity

8. Apply a polynomial solution to the differential equation of equilibrium to the problem of straight bar fixed at one end and bending due to a load P applied at the other end. Obtain the deflections at loaded end.

9. Obtain stress distribution in a rotating disk

10. Evaluate the effect of a circular hole on stress distribution in infinite plate subjected to uniform tension in one direction.

11. Derive the saint venants solution to the problem of Torsion in straight bars and apply this solution to a bar with circular cross section.

12. Evaluate the stress distribution and displacements in prismatic bar subjected to pure bending treating it as a 3 – D problem

13. Explain membrane analogy this analogy to evaluate stress distribution under Torsion of a Bar of Narrow rectangular cross section.

14. Based on saint venants solution for Torsion evaluate the shear stress distribution in a cantilever loaded at the free end and having a circular cross section.

15. Derive the differential equation of equilibrium for 2 – D problem of elasticity

1. Apply a polynomial solution to the differential equation of equilibrium to the problem of straight bar fixed at one end and bending due to a load P applied at the other end. Obtain the deflections at loaded end.

2. Evaluate stresses in a simply supported beam cross section where a udl of q/m is acting on the beam. Also calculate maximum deflection.

3. Using a general solution to the differential equation of equilibrium in polar coordinates, calculate stresses and deflections in a circular disc with a whole at the centre.

4. Obtain stress distribution in a rotating disk

5. Evaluate the effect of a circular hole on stress distribution in infinite plate subjected to uniform tension in one direction.
1. Derive the 4th order differential equation of equilibrium for a rectangular plate by explaining moment curvature relationships.
2. Obtain Navier Solution to the deflections and moments in a SS rectangular plate with a uniformly distributed lateral load.
3. Evaluate the LEVY solution for deflections to a rectangular plate with opposite edges clamped.
4. Apply a general solution to the equilibrium equation of a circular plate to a SS circular plate.
5. Obtain Navier Solution for SS Rectangular plate with pointload using strain energy formulation for deflection of plates.
6. Obtain deflection & moments in a circular plate with a hole at centre SS on outer edge and uniformly loaded.

1. Obtain the strain – displacement relation
2. Derive the D.E of equilibrums interms of displacement components
3. a) Explain the advantages of stress tensor and strain tensor.
   b) Explain plane stress and plane strain with examples
   c) What is meant by equilibrium and compatibility conditions.
4. Considering the plane strain derive the D.E of compatibility without body forces.
5. The state of stress at a point with respect to x,y,z system is

\[
\begin{pmatrix}
10 & 5 & -15 \\
5 & 10 & 20 \\
-15 & 20 & 25 \\
\end{pmatrix}
\text{kN/sq.m}
\]

Determine the stress relative to \(x^1, y^1, z^1\) coordinate systems obtained by a rotation through 45°, about Z axis
7. Investigate what problem of plane stress is satisfied by the stress function.
   \(\Phi = 3f /4d \ (xy-xy^3/3d^3) + py^2/2\)
   Applied in the region \(y = 0, y = d, x = 0\)
8. a) What are the advantages of fourier series
   b) Obtain the equation of stress function by fourier series.
9. Obtain the strain – displacement relation
10. Derive the D.E of equilibrums interms of displacement components
11. a) Explain the advantages of stress tensor and strain tensor.
    b) Explain plane stress and plane strain with examples
    c) What is meant by equilibrium and compatibility conditions.
12. Considering the plane strain derive the D.E of compatibility without body forces.
13. The state of stress at a point with respect to x,y,z system is

\[
\begin{pmatrix}
10 & 5 & -15 \\
5 & 10 & 20 \\
-15 & 20 & 25 \\
\end{pmatrix}
\text{Kn/sq.m}
\]

Determine the stress relative to \(x^1, y^1, z^1\) coordinate systems obtained by a rotation through 45°, about Z axis
15. Investigate what problem of plane stress is satisfied by the stress function.
   \(3f(xy-xy^3) + py^2\)
   Applied in the region \(y = 0, y = d, x = 0\)
16. a) What are the advantages of fourier series
    b) Obtain the equation of stress function by fourier series.

1. Obtain the equilibrium equation in 2 D – problems in polar coordinates.
2. For a hallow cylinder under uniform pressure obtain the radial, circumferential and longitudinal stresses.
3. Obtain the stresses distribution with the effect of circular hole in a plate.
4. Explain max well bettis and castigianos’s theorems for stresses.
Previous Examination Questions

5. Derive the D.E for bending of a cantilever by terminal loads with (i) circular section and (ii) with elliptical section.
6. Draw the stress distribution for torsion of elliptical cross section.
7. For an elastic body explain the following using stress and strain components in three dimensions.

1. Principal stresses and stress ellipsoid
2. Explain STRESS Invariants and determine principal stress and max shearing stresses for the following stress state.
   
   \[
   \sigma_x = 4 \, \text{N/mm}^2 \\
   \sigma_y = 2.5 \, \text{N/mm}^2 \\
   \sigma_z = 1 \, \text{N/mm}^2
   \]
3. Explain strain energy formulation.
4. Explain homogeneous deformation and rotation
5. Derive using St. Venants semi inverse method the stress function for Torsion of non circular shafts and obtain Twisting moment in term of this stress function. Hence apply this to an elliptic c/s and obtain distribution of shear stresses in a c/s.
6. Explain membrane analogy and derive its formulation for Torsion of non circular shafts. Hence obtain solution in terms of shear stresses in a bar of Narrow rectangular cross section subject6ed to Twisting moment.
7. Explain briefly
   a. Torsion of Rolled profile sections.
   b. Torsion of Hollow shafts.
   c. Torsion of Thin tubes.
8. Obtain displacements in a prismatic bar subjected to pure bending.

1. Derive the differential equation of equilibrium in term of stress for a 2 – D problem of Elasticity and write the general polynomial form of solution to the above different equations?
2. Evaluate the displacements of a cantilever beam subjected to a point load at free end?
3. From the general solution of symmetric stress distribution problem in polar coordinates derive the stresses in the case of pure bending of curved bar?
4. Explain the stress distribution in rotating disk and the effect of a hole at the center of disk?
5. How does a circular hole effect the stress distribution in a plate under uniform stress distribution. Explain and sketch the distribution?
6. If an infinite large plate is loaded at the straight boundary with a concentrated point load. Derive the radial solution for the stress distribution in the plate. sketch the variation of stresses?

1. Derive the differential equation for equilibrium and compatibility in term of stress for a 2 – D problem of Elasticity and write the general polynomial form of solution to the above differential equations?
2. Apply a general polynomial solution of governing differential equation to the case of bending of cantilever loaded at the end, and obtain stresses, strains and displacements.
3. Write a general solution for a problem in polar coordinates when stress distribution is symmetrical about an axis. Hence obtain stresses for a circular plate with a hole at centre.
4. Obtain a solution (stress component and displacements) to the problem of rotating disk
5. How does a circular hole effect the stress distribution in a plate under uniform Stress distribution. Explain and evaluate the distribution and sketch the results
6. If an infinite large plate is loaded at the straight boundary with a concentrated point load. Derive the radial solution for the stress distribution in the plate. sketch the variation of stresses on a horizontal plane.

1. Apply a general polynomial solution of governing differential equation to the case of bending of cantilever loaded at the end, and obtain stresses, strains and displacements.
2. From the general solution of symmetric stress distribution problem in polar coordinates derive the stresses in the case of pure bending of curved bar?
3. Explain the stress distribution in rotating disk and the effect of a hole at the center of disk?
4. How does a circular hole effect the stress distribution in a plate under uniform stress distribution? Explain and sketch the stress distribution?

5. If an infinite large plate is loaded at the straight boundary with a concentrated point load. Derive the radial solution for the stress distribution in the plate. Sketch the variation of stresses?

6. For the following stress tensor generate a stress ellipsoid and obtain principal stresses principal planes and hence formulate the stress invariants

\[
\begin{bmatrix}
20 & 16 & 10 \\
16 & 30 & 12 \\
10 & 12 & 15
\end{bmatrix}
\]

1. Apply a polynomial solution to the differential equation of equilibrium to the problem of straight bar fixed at one end and bending due to a load P applied at the free end. Obtain the deflections at loaded end.

2. Evaluate the effect of a circular hole on stress distribution in infinite plate subjected to uniform tension in one direction.

3. Evaluate stresses in a simply supported beam cross section where a udl of q/m is acting on the beam. Also calculate maximum deflection.

4. Obtain stress distribution in a rotating disk

5. a) Write the equation of equilibrium in terms of displacements and hence write general solution to differential equation.
   b) Determine displacements by writing strain displacement solution and hence obtain general form of displacements that include rigid body displacements.

6. Explain membrane analogy. Apply this analogy to the problem of bar of narrow rectangular section subjected to torsion and calculate the stress.

7. Explain briefly
   (i) Stress invariance
   (ii) Stress ellipsoid
   (iii) Principal stress & principal planes
   (iv) Homogeneous deformation

8. Derive the saint venants solution to the problem of Torsion in straight bars and apply this solution to a bar with circular cross section.

1. Obtain the Governing D.E for two dimensional problem in polar coordinates using compatibility.

2. a) Obtain the expressions for strain components in polar coordinates.
   b) Obtain the stress components for a thin rotating hallow disk.

3. Using general theorems obtain the expression for condition of compatibility

4. Explain Maxwell’s bettis and castiglianos theorems.

5. obtain the displacements in bending of prismatic bar subjected to pure bending

6. Explain saint venant torsion for elliptical cross section and torsion of thin walled tubes.

1) a) obtain strain displacement relations
   b) Derive the Differential equation of equilibrium for plane stress neglecting body forces

2)a) What is meant by compatibility and obtain the condition for compatibility
   b) Considering plane strain problem obtain the expression for compatibility in terms of stresses.

4) a) Explain airy’s stress function by considering body force
   b) Explain plane stress and plane strain

5) Given the following stress function
   \[ \Phi = -Fx/y^2 \ (3d-2y) \]
   determine the stress components and sketch them

6) A cantilever beam of uniform cross section is subjected to a point load p at its end. Determine the constants C1,C2,C3 if the stresses are \( \sigma_x = C_1 xy \), \( \sigma_y = 0 \); \( T_{xy} = C_2 + C_3 y^2 \). Also determine the strain components and find whether these are compatible or not
   Boundary Conditions are \( T_{xy} \) at \( y = \pm C = 0 \)
   \( \int T_{xy} \ dy = -p \)
   \( \int \sigma_x \ y \ dy = - px \)
1. Derive the governing differential equation for circular plates?
2. Obtain expression for deflection for a circular plate with a central-hole bent by moments M1 & M2 uniformly distributed along inner and outer boundaries?
3. Derive the governing differential equation for bending of isotropic plates?
4. Derive the governing differential equation for plates subjected to lateral loading and in-plane forces?
5. Using finite difference techniques, find the maximum deflection and bending moment for a square plate (a x a) loaded with udl of intensity ‘P’ if the plate is fixed at the edges? (consider $\alpha =a/2$ and $\gamma =0.3$).
6. Find the maximum deflection for a square plate fixed at edges and loaded with udl of intensity ‘P_o’, using Galerkin’s method? Take poisson’s ratio as 0.3.
****Previous Examination Questions****

(a) Explain about plane stress and plane strain problems. Give two examples also.

(b) Derive the compatibility equation in terms of stress for a plane stress problem. Is this equation valid for plane strain also?

(c) The general displacement field in a body in certain coordinates is given as:
\[ u = 0.015x^2y + 0.03 \]
\[ v = 0.005y^3 + 0.03xz \]
\[ w = 0.003x^2 + 0.001yz + 0.005 \]
Find all the strains for the point (1,0,2)

2 Attempt any two parts of the following : 10x2=20

(a) Derive the expression for circumferential stress in a curved beam with large initial armature and subjected to pure bending. State clearly the assumptions and its limitations.

(b) A circular plate with a circular hole is simply supported around its edge and subjected to linearly varying distributed load. Derive the expressions for maximum stress.

(c) A narrow, simply supported beam of rectangular cross-section is subjected to a uniformly distributed load. Determine the stress distribution in the beam.

3 Attempt any one part of the following : 20x1=20

(a) Determine the distribution of stress is a circular cylindrical shell having the ends supported by the diaphragms. The shell has been filled with oil of density \( P \) such that
\[ P(Q) = 10^6 \text{ Pa} \cos Q \]
Where \( a \) = radius

(b) Derive the expressions for the stress resultants and displacements for the case of a cylindrical shell with a uniform pressure.

4 Attempt any one part of the following : 20x1=20

(a) Derive an expression for strain energy per unit volume for a two-dimensional linearly elastic body for plane stress or plane strain in terms of Airy’s stress functions.

(b) How do you determine the stress distribution due to cracks? Explain with a suitable example.
1. The stress components at a point are $\sigma_x = 100, \sigma_y = 50, \sigma_z = 40, \tau_{xy} = 20,$ $\tau_{yz} = -40, \tau_{zx} = -60$ MPa. Determine the resultant stress on a plane whose direction cosines are $(1/3, -2/3, 2/3)$.

2. The displacement components are given by the relations $u = x - 2y, v = 2x + 2y, w = 5z$. Show that the displacement vector is physically possible for a continuously deformed body.

3. What do you mean by inverse method in elasticity?

4. Determine the radial and shear stresses for the Airy's stress function, $\psi = \cos^2 \theta$.

5. Show that $\nabla^2 \psi = 0$ where $\psi$ is, St. Venant's warping function.

6. A hollow tube 50 mm mean diameter and 2 mm wall thickness with a 2 mm wide saw cut along its length is subjected to a twisting moment. If the maximum shear stress induced is 5 N/mm$^2$, find the value of the twisting moment.

7. State Engesser's theorems.

8. Write the expression of finding displacement at any section of a loaded beam using the principle of virtual force.


10. Explain soap film analogy for plastic torsion.
11. (i) Derive the Navier's equilibrium equation in Cartesian coordinates in terms of displacements.

(ii) The stress components at a point are given by \( \sigma_x = 200, \sigma_y = -240, \sigma_z = 160, \tau_{xy} = 160, \tau_{yz} = 100, \tau_{zx} = -120 \) N/mm². Determine the normal strain components at this point. Assume the modulus of elasticity and Poisson’s ratio of the material as 210 kN/mm² and 0.3 respectively.

12. (a) Show that \( \phi = \frac{q}{8c^3} \left[ x^2(y^3 - 3c^2y - 2c^3) - \frac{1}{5} y^3(y^2 - 2c^2) \right] \) is a stress function and find what problems it solves when applied to the region included in \( y = \pm C, x = 0 \) on the side \( x \) positive.

Or

(b) Derive the expression for stress components in a thin plate of infinite dimension with a central circular hole under uniform uniaxial tension.

3. (a) Derive the expression for the angle of twist, shear stress at any point and hence maximum shear stress in a bar of elliptical section due to a twisting moment.

Or

(b) A thin walled box section of dimensions \( 2a \times a \times t \) is to be computed with a solid section of diameter \( d \) (fig. Q. 13 (b)). Find the thickness so that two sections have

(i) the same maximum shear stress for the same torque

(ii) the same stiffness.

\[ \text{Fig. Q. 13 (b)} \]
14. (a) Determine the expression for the total strain energy in terms of components of stress and strain.

Reduce the above expression for the case of (i) plane stress (ii) simple tension (iii) symmetrical bending and (iv) torsion.

Or

(b) Using Rayleigh Ritz method, find the critical load of a long column fixed at one end and free at the other end.

15. (a) (i) What do you understand by yield criteria? (4)

(ii) A thin walled tube of mean radius 100 mm and wall thickness 4 mm is subjected to a torque of 10 N-m. If the yield strength of the tube materials is 120 N/mm², determine the value of the axial load applied to the tube so that the tube starts yielding according to the Von Mises criteria. (12)

Or

(b) (i) What is meant by residual stress with respect to torsion? (4)

(ii) A solid circular shaft of 100 mm radius is subjected to a twisting moment so that the outer 50 mm deep shell yields plastically. If the yield stress in shear for the shaft material is 175 N/mm², determine the twisting couple applied and the associated angle of twist. Assume the shear modulus of the shaft material as 84 kN/mm². (12)
1. Derive Equilibrium Equations for a 3 Dimensional State of Stress?

2. The state of stress at a point is given by
   \[ \sigma_{xx} = 10, \quad \tau_{XY} = 8 \]
   \[ \sigma_{YY} = -6, \quad \tau_{YZ} = 0 \]
   \[ \sigma_{ZZ} = 4, \quad \tau_{ZX} = 0 \]

   Consider another set of Co-ordinate axis \( X', Y', Z' \) in which \( Z' \) coincides with \( Z \)-axis and \( X' \) is rotated by 30° anticlock wise from the \( X \) axis. Determine the stress components in the new system.

3. Derive Equilibrium & Compatibility equations for a body in polar co-ordinate system?

4. What is plane strain & plane stress problems? Explain with an example and derive appropriate equations for the above problems?

5. By assuming appropriate stress function “\( \Phi \)”, derive deflection equation for a simply supported beam carrying a u.d.l of \( q \) kN/m.

6. Calculate the Torque carrying capacity for an elliptical cross section by stress function approach?

7. What is membrane analogy? By Membrane analogy calculate the Torsion in Circular body?

8. Explain in Detail the Following yield criteria with neat Sketches?
   a) Maximum Shear Criteria
   b) Distortion Energy Criteria
1. Define the terms:
   (a) Homogeneous
   (b) Isotropy.

2. Write down the partial differential equation of equilibrium in polar coordinate system.

3. Mention a practical example for plane stress and plane strain problem.

4. Write the biharmonic equation in Cartesian system used to solve a torsional problem in semi-inverse approach.

5. Give the concept of membrane analogy.

6. Express the maximum shear stress and angle of twist per unit length of a thin rectangular section of size $b \times d$.


8. List the various energy theorems.

9. What is strain hardening?

10. Define yield criteria.
11. (a) The state of stress at a point in a strain material are given by the following array, \[
\begin{bmatrix}
9 & 15 & 24 \\
15 & 1 & 0 \\
24 & 0 & 2
\end{bmatrix}
\] \(N/mm^2\). Determine the principle stresses and the associated direction cosines.

Or

(b) The state of stress at a point for a given reference axis xyz is given by the following array of terms. The stresses are in MPa.
\[
\begin{bmatrix}
60 & 30 & -20 \\
30 & 30 & 25 \\
-20 & 25 & 20
\end{bmatrix}
\]

(i) Determine the stress invariants

(ii) If a set of new axes \(xy'z'\) is formed by rotating about the \(z\) axis in anticlockwise direction by \(45^\circ\), determine the stress components in the new coordinate system.

12. (a) Show that in the absence of body forces the displacements in problems of plane stress must satisfy:
\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{(1+v)}{1-v} \frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] = 0.
\]

Or

(b) A stress function is given by \(\phi = -\frac{2P}{d^3b} xy^2 + \frac{3Pxy}{2bd} + K_x x + K_z\). Show that stress function \(\phi\) solves the problem of a cantilever beam with a rectangular cross section and a concentrated load at free end.

13. (a) (i) Explain the St. Venant's method to solve torsional problems.

(ii) A bar of circular section \(f(x, y) = x^2 + y^2 - a^2 = 0\) is twisted by torque \(T_x\). Investigate the state of stress in the bar using a suitable stress function using St. Venant's method.

Or
(b) A hollow multi-cells aluminium tube of cross section as shown in Fig. Q.13(b) resist a torque of 5kN-m. The wall thickness are $t_1 = t_2 = t_4 = t_5 = 0.5 \text{ mm}$, $t_3 = 0.75 \text{ mm}$. Determine the maximum shear stress and angle of twist per unit length. Take $G = 25 \text{ GPa}$. All dimensions in the figure are in ‘m’.

Fig. Q.13(b)

14. (a) Explain the strain energy of a 3D stress system by applying to an elastic body subjected to stresses $\sigma_1, \sigma_2$, and $\sigma_3$ [principal stresses].

Or

(b) Explain in detail about the principle of virtual work. Also discuss about the applications.

15. (a) A steel bolt is subjected to a bending moment of 240 kN-m and torque 140 kN-m. If the yield-stress in tension for the bolt material is 250 MPa, find the diameter of the bolt, according to (i) Tresca’s (ii) Von Mises.

Or

(b) A member is subjected to design loads. The calculated stresses are $\sigma_x = 80 \text{ MPa}$, $\sigma_y = 240 \text{ MPa}$, $\tau_{xy} = -80 \text{ MPa}$. The yield stress of material is $\sigma_y = 500 \text{ MPa}$. Determine the factor of safety as per (i) Tresca criteria and (ii) Von Mises Criteria.
1. Explain strain tensor.
2. Octahedral stresses.
3. Write short notes on Prandtl's membrane analogy.
4. Explain briefly about St.Venant's Approach for torsion.
5. Write down polynomial of the second degree.
6. Define stress concentration factor.
7. Define Winkler's constant.
8. Compare Kelvin's and Boussinesq's solutions.
9. State Von-Mises criterion
10. Write the final equation for plastic stress-strain relationship.

PART B — (5 × 16 = 80 marks)

11. (a) The state-of-stress at a point is given by the following array of terms

\[
\begin{bmatrix}
9 & 6 & 3 \\
6 & 5 & 2 \\
3 & 2 & 4 \\
\end{bmatrix}
\text{MPa.}
\]

Determine the principal stresses and principal directions.
(b) The components of strain at a point is given by
\[ \varepsilon_x = 0.15, \varepsilon_y = 0.25, \varepsilon_z = 0.40, \gamma_{xy} = 0.10, \gamma_{xz} = 0.15, \gamma_{yz} = 0.20. \]

(i) If the coordinate axis are rotated about z axis through 60 degree in the anticlockwise direction determine the new stress components.

(ii) Also find principal stress and its orientation.

12. **(a)**
   (i) Discuss the effect of radial and tangential stress for a circular hole on a plate. 
   
   (ii) Find the expression for normal and shear for a circular disc subjected to compression along the diameter.

   (8)

Or

(b) Show that the following stress function satisfies the boundary condition in a beam of rectangular cross-section of width 2h and depth d under a total shear force W.
\[ \phi = \frac{W}{2nd^2} xy^2(3d - 2y). \]

13. **(a)**
A thin walled steel section shown in figure 1 is subjected to a twisting moment T. Calculate the shear stresses in the walls and the angle of twist per unit length of the box.

Figure – 1

Or

(b) Discuss the effect of shear and torsion on (i) elliptical cross section and (ii) triangular cross section of bar. 

(8+8)

14. **(a)**
Find out bending moment and shear force for Semi-Infinite beams with concentrated loads.

Or

(b) Find out bending moment and shear force for Infinite beams with concentrated loads.
15. (a) (i) A steel bolt is subjected to a bending moment of 300 Nm and a torque of 150 Nm. If the yield stress in tension for the bolt material is 250 MPa, determine the diameter according to (i) Tresca criteria and (ii) Von-Mises criteria.

(ii) A cantilever beam 10cm wide, 12cm deep is 4m long and is subjected to an end load of 500 kg. If the stress curve for the material is given by $\sigma = 7000(\varepsilon)^{0.2}$ (in kg cm unit) determine the maximum stress method and the radius of curvature.

Or

(b) The state of stress at a point is given by $\sigma_x = 70$ MPa, $\sigma_y = 120$ MPa and $\tau_{xy} = 35$ MPa, if the yield strength for the material is 125 MPa, check whether yielding will occur according to Tresca’s and Von Mises condition.
***Previous Examination Questions***

1. Derive the equations of equilibrium for a 3-D stress state. (10 Marks)

2. A point P in a body is given by
   \[
   Z = \begin{bmatrix}
   100 & 100 & 100 \\
   100 & -50 & 100 \\
   100 & 100 & -50
   \end{bmatrix}
   \text{mN/mm}^2
   \]
   Determine the total stress, normal stress and shear stress on a plane which is equally inclined to all the three axes. (10 Marks)

3. a. What is meant by stress invariants? With a sketch show that stress invariants are the same. (10 Marks)
   b. The state of stress at a point is characterized by
   \[
   Z = \begin{bmatrix}
   12 & -3 & 0 \\
   3 & 4 & 0 \\
   0 & 0 & 10
   \end{bmatrix}
   \text{MPa}
   \]
   Determine the principle stresses and directions for any principal stress. (10 Marks)

4. a. Derive the compatibility relation of strain in a 3-D elastic body. What is its significance? (10 Marks)
   b. The state of stress at a point is given by
   \[
   \sigma_x = 200 \text{ MPa}, \quad \sigma_y = -100 \text{ MPa}, \quad \sigma_z = 50 \text{ MPa}
   \tau_{xy} = 40 \text{ MPa}, \quad \tau_{xz} = 50 \text{ MPa}, \quad \tau_{yz} = 60 \text{ MPa}
   \]
   If \( E = 2 \times 10^5 \text{ N/mm}^2 \) and \( G = 0.8 \times 10^5 \text{ N/mm}^2 \). Find out the corresponding strain components from Hook’s law. Take \( \gamma = 0.2 \). (10 Marks)

5. a. Derive the stress components for a plate with circular hole subjected to an uniaxial load. (10 Marks)
   b. Derive the equilibrium equation in cylindrical coordinates for 2-D elastic body. (10 Marks)

6. a. Starting from the fundamentals derive the expression for hoop and radial stresses for a rotating hollow disc. (10 Marks)
   b. Show that \( M_e = GJ\theta \) in torsion of shafts with usual notations. Where \( G \) – modulus of rigidity \( J \) – polar moment of inertia and \( \theta \) – angular twist for unit length. (10 Marks)

7. a. Write the thermo elastic stress-strain relationships for 3-D elastic body. (10 Marks)
   b. Derive the thermal stresses in a thin circular disc. (10 Marks)

8. Write a short notes on:
   a. Saint – Venants principle
   b. Plane stress and plane strain
   c. Principle of super – position
   d. Membrane analogy. (20 Marks)
1. Define body forces with examples.

2. State any two examples each for plane stress and strain problems.

3. What is Airy's stress equation?

4. Write the polynomial equation for first and second degree functions if \( \varphi = a_2 x + b_2 y \).

5. State the membrane analogy for torsion.

6. Define warping torsion.

7. Define virtual work.

8. What is strain energy?

9. What is plastic potential?

10. State the assumptions in yield criteria.
11. (a) (i) The strain components at a point are given by

\[ \varepsilon_x = 10xy + 12z \quad \varepsilon_y = 4xy^2 \]
\[ \varepsilon_z = 5xy^2 + 2yz \quad \varepsilon_y = 2yz^2 \]
\[ \varepsilon_z = 2x^2z + 2y \quad \varepsilon_z = 2xz^2 \]

Verify whether the compatibility equations are satisfied or not. (6)

(ii) The strain components at a point are given by

\[
\begin{bmatrix}
10 & 15 & 20 \\
15 & 25 & 15 \\
15 & 16 & 30 \\
\end{bmatrix}
\text{MPa}
\]

If the system is rotated by 45° about the z-axis in the anticlockwise direction, find the new stress tensor. (10)

Or

(b) (i) Explain generalized Hooke's law. (6)

(ii) Derive the equations of equilibrium and compatibility conditions in Cartesian co-ordinates for a two-dimensional stress field. (10)

12. (a) The stress tensor at a point is given by

\[
\begin{bmatrix}
10 & 6 & -12 \\
6 & 16 & 9 \\
-12 & 9 & 21 \\
\end{bmatrix}
\text{MPa}
\]

Determine the principal stresses and principal planes. (16)

Or

(b) Apply the stress function \( \Phi = -\left( \frac{F}{2hd^2} \right)xy^2(3d-2y) \) on a beam of rectangular section of breadth '2a' and depth 'd'. Determine what kind of problem is solved by this stress function. Is the solution perfect or imperfect? Comment on the results. (16)
13. (a) A closed thin walled tube has a perimeter ‘L’ and a uniform wall thickness ‘A’. An open tube is made by making fine silt in it. Show that when the maximum shear stress is the same in both closed and open tubes,

\[
\frac{T_{\text{open}}}{T_{\text{closed}}} = \frac{LH}{6A} \quad \text{and} \quad \frac{\theta_{\text{open}}}{\theta_{\text{closed}}} = \frac{2A}{LH}
\]

where A is the silt. (16)

Or

(b) (i) Obtain the St. Venant’s torsion equation and state how will you obtain the shear stresses and angle of twist. (7)

(ii) A thin walled box section having dimensions $200 \text{ mm} \times 100 \text{ mm} \times t' \text{ mm}$ is to be compared with a solid circular section of diameter 100 mm. Determine the thickness ‘t’ so that the two sections have

1. Same maximum shear stress for the same torque
2. The same stiffness. (9)

14. (a) A square bar of cross section $60 \text{ mm} \times 60 \text{ mm}$ is subjected to a twisting moment of $180 \text{ Nm}$ at the end. $G = 80 \text{ GPa}$. Find the maximum shear stress and the angle of twist per unit length. Adopt strain energy method and proceed from fundamentals. (16)

Or

(b) (i) Briefly discuss Rayleigh-Ritz method. (5)

(ii) Assuming a suitable equation for the deflection curve, determine the deflection of a cantilever beam of span ‘$T$’, carrying a concentrated load ‘$P$’ at the free end. (11)
14. (a) Compute the first three natural frequencies and the corresponding mode shapes of the transverse vibrations of a uniform beam if the ends are simply supported. Proceed from fundamentals and derive any equation that you may adopt.

Or

(b) (i) For a cantilever beam with mass and stiffness matrices as given below, determine the fundamental frequency by Rayleigh’s method.

\[
\begin{bmatrix}
    m & 0 & 0 \\
    0 & m & 0.5m \\
    0 & 0.5m & m
\end{bmatrix};
\begin{bmatrix}
    2K & -K & 0 \\
    -K & 2K & -K \\
    0 & -K & K
\end{bmatrix}
\]

(ii) Determine the first two modes of the above problem by Rayleigh-Ritz method by assuming,

\[
\begin{bmatrix}
    1.00 \\
    0.670 \\
    0.200
\end{bmatrix}
\]

15. (a) Describe briefly how will you idealise and formulate a structure subjected to blast loading.

Or

(b) Write short notes on the following:

(i) Deterministic analysis of Earthquake

(ii) Gust phenomenon.
1. (i) Let \( x_1, x_2, x_3 \) be rectangular Cartesian co-ordinates and \( \theta_1, \theta_2, \theta_3 \) be spherical polar co-ordinates having the following relationship:

\[
x_1 = \theta_1 \sin \theta_2 \cos \theta_3; \quad x_2 = \theta_1 \sin \theta_2 \sin \theta_3; \quad x_3 = \theta_1 \cos \theta_2
\]

Get the components of Euclidian Metric tensor and the length of the line element. (12)

(ii) What do you understand by Cauchy’s Stress Ellipsoid? Explain. (8)

2. (i) Derive the relation between the Lamé’s Coefficient and the elastic constants. (10)

(ii) State the conditions under which the following is the possible system of strains:

\[
\begin{align*}
\varepsilon_{xx} &= a + b (x^2 + y^2) + x^4 + y^4 \\
\varepsilon_{yy} &= a + b (x^2 + y^2) + x^4 + y^4 \\
\gamma_{xy} &= A + Bxy (x^2 + y^2 - C^2) \\
\gamma_{yz} &= 0; \quad \gamma_{xz} = 0; \quad \varepsilon_{zz} = 0
\end{align*}
\]

(10)

3. As a result of measurements made on the surface of a machine component with strain gages oriented in various ways, it was established that the principal strains on the free surface are \( \varepsilon_a = +400 \times 10^{-6} \); \( \varepsilon_b = -50 \times 10^{-6} \).

(i) Calculate the value of maximum in plane shearing strain.

(ii) Find absolute maximum shearing strain for the system (Given that \( \sigma_c = 0 \) for the free surface and Poisson ratio, \( \nu = 0.3 \)). (20)

4. (i) Explain the development of Tresca Yield criteria. (10)

(ii) Write a short note on Plastic stress – strain relations. (10)
5. A state of plane stress shown in figure occurs at a critical point of a steel machine component.

\[
\begin{array}{c}
25\text{MPa} \\
\downarrow \\
20\text{MPa} \\
\downarrow \\
80\text{MPa} \\
\downarrow \\
40\text{MPa} \\
\end{array}
\]

(i) Determine whether the machine will fail or not if the tensile yield strength is \( \sigma_y = 250\text{MPa} \) for the grade of steel used by using maximum shearing stress criteria.

(ii) Determine the factor of safety with respect to yield using both the maximum shearing stress criteria and maximum distortion energy criteria. (20)

6. (i) What is a Viscoelastic material. Explain the different ways to model its behaviour. (10)

(ii) Explain the true Stress – strain curve for a ductile material. Also, illustrate the influence of Bauschinger Effect, strain rate and temperature on the curve. (10)
******Previous Examination Questions******

1.a) Derive the boundary conditions in Cartesian coordinates of a three dimensional system.

b) Determine the principal stresses, maximum shear stress, octahedral normal and shear stress at a point
\[ \sigma_x = 4 \text{MPa}, \sigma_y = 8 \text{MPa}, \sigma_z = -12 \text{MPa} \]
\[ \tau_{xy} = \tau_{yz} = 0, \tau_{xz} = 2 \text{MPa} \]

2.a) Determine the principal strain and principal plane for the given state of strain
\[ \varepsilon_x = 0.1, \varepsilon_y = -0.05, \varepsilon_z = -0.05, \gamma_{xy} = 0.3, \gamma_{yz} = 0.1 \text{ and } \gamma_{xz} = -0.08 \]

b) Write down the strain transformation formula. The state of strain is given by
\[ \varepsilon_x = -200(10)^{-6}, \varepsilon_y = -0.05, \varepsilon_z = 0.05, \gamma_{xy} = 0.3, \gamma_{yz} = 0.1 \text{ and } \gamma_{xz} = -0.08 \]

Determine the strain in another set of axis if the X axis is rotated 30 degrees in the clockwise direction.

3.a) What are the compatibility conditions? Derive the compatibility conditions in terms of strains.

b) Prove that \((\lambda + 2G)\varepsilon^2 = 0\).

4. Derive the elastic curve expression of a cantilever subjected to a point load at the free end.

5.a) Derive the equilibrium equations in polar coordinates.

b) A thick cylinder is subjected to internal pressure. Prove that the circumferential stress is numerically greater than the internal pressure in the inner surface of the cylinder.

6.a) Derive the equilibrium equation and boundary he of a bar subjected to pure torsion.

b) Explain membrane analogy applied to narrow rectangular sections and derive the torsional constant and maximum shear stress for a narrow rectangle.

7.a) Discuss the yield criteria and flow rules for perfectly plastic and strain hardening materials.

b) Discuss the elasto plastic analysis for a beam subjected to torsion.

8. Write short notes on the following:
   a) Plane stress problem and plane strain problem.
   b) Reciprocal theorem.
   c) Principle of superposition.
   d) Saint Venants principle.
1. (a) Derive equations of equilibrium for 3-D cartesian system of coordinates. 8

(b) Derive strain-displacement relationships for 3-D cartesian system of coordinates. 12

2. Derive the expressions for finding out radial stress, tangential stress and shear stress on a large plate with a small hole when subjected to direct tensile stress, $s$ (uniaxial). 20

3. Stress tensor at a point is given by:

$$\sigma_{ij} = \begin{pmatrix} 10 & 15 & 20 \\ 15 & 25 & 15 \\ 20 & 15 & 30 \end{pmatrix}.$$
Find out:

(i) Principal stresses and their directions.  

(ii) Maximum and minimum shear stresses alongwith their planes.

4. Find out stresses in a cantilever beam by Airy's stress function approach when it is subjected to a point load at the free end. The width of the beam is \( h \) and depth of the beam is \( d \).

5. A rectangular beam 8 cm wide and 10 cm deep is 2 m long and is simply supported at the ends. The yield strength of the material is 250 MPa. Determine the value of the concentrated load applied at the midspan of the beam if (a) the outermost fibres of the beam just start yielding, (b) the outer shell upto 3 cm depth yielded, and (c) whole of the beam yielded. Assume the material is linearly elastic and perfectly plastic.

6. A solid circular shaft of 10 cm radius is subjected to a twisting couple so that the outer 5 cm deep shell yields plastically. If the yield strength in shear for the shaft material is 175 MPa, determine the twisting couple applied and the associated angle of twist. \( G = 0.84 \times 10^5 \) N/mm\(^2\).

7. A thick cylinder of internal radius 15 cm and external radius 25 cm is subjected to an internal pressure \( p \)
MPa. If the yield strength of the cylinder material is 240 N/mm², determine (a) pressure at which the cylinder will start yielding just at inner radius, (b) the stresses when the cylinder has a plastic front of radius 20 cm, and (c) stresses when whole of the cylinder has yielded.

Assume Tresca yield criterion and plane strain condition. 20

3. A thin circular disc of uniform thickness is of 50 cm outer diameter and 20 cm inner diameter. Determine (a) speed of rotation so that the disc just starts yielding plastically at the inner radius, (b) stresses in the disc when disc has yielded upto 15 cm radius and (c) the speed for full yielding. Given: \( \rho = 7850 \) kg/m³, \( \sigma_y = 250 \) N/mm² and \( v = 0.30 \). 20
1. (a) Define surface force and body force.
   (b) Define plane stress in (3-D) system.
   (c) Define plane strain in (3-D) system.
   (d) Define stress in (2-D) and (3-D) system  \( \text{(4 x 5)} \)

**SECTION A**

2. (a) Prove that shear stress \( \tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx} \) and \( \tau_{yz} = \tau_{zy} \).
   (b) Derive a relationship between Bulk modulus (K) and modulus of elasticity (E).
   (c) Define stress function (φ).
   (d) Derive the differential equation of equilibrium of (3-D).  \( \text{(4 x 5)} \)

3. (a) Prove lame stress ellipsoid in three dimensional system.
   (b) Show that plane strain case is reduce to plain stress case.
   (c) Prove Hooks law in three dimension system.
   (d) Derive a relationship between shear modulus (G) and modulus of elasticity (E).  \( \text{(4 x 5)} \)

4. An elastic layer sandwiched between two perfectly rigid plate to which it is bounded. The layer is compressed between the plates in such a way that the attachments to the plates prevent lateral strain completely. Find the apparent modulus of elasticity and apparent Poisson’s ratio. Also prove that the apparent modulus of elasticity is many times of the actual modulus of elasticity if Poisson’s ratio is slightly less than 0.5.  \( \text{(20)} \)

**SECTION B**

5. Obtain the compatibility equation for plane strain case.  \( \text{(20)} \)

6. The state of stress at a point for a given reference is given below as \( \tau_{ij} \).
   If a new set of axes is formed by rotating \( \text{xyz} \) through 45° about \( z \)-axis. Find the new stress tensor \( \tau_{ij} \):
   \[
   \tau_{ij} = \begin{pmatrix}
   300 & 100 & 0 \\
   100 & 200 & 0 \\
   0 & 0 & 105
   \end{pmatrix}
   \]
   \( \text{(20)} \)

7. For the given function (φ)
   \[\phi = \frac{F}{d^2} x y^2 (3d-2y)\], determine the stress component.  \( \text{(20)} \)